ON CONTACT ROLLING OF ELASTIC CYLINDERS

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In his dissertation* Glagolev investigated the problem of contact rolling of cylinders of similar materials. He established that, in the absence of the traction force T (see schematic diagram below), the shear forces acting on the contact surface are vanishing and that the state of stress in this case is identical with simple compression of elastic bodies.

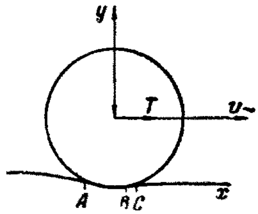

Fig. 1.

If the traction T is present, the contact curve is divided by the point C into two parts: the adhesion region, AC, and the slip region CB. The total "slipping" is determined from

$$\delta = V\frac{ka}{R}\left[\sqrt{1 + T/pk} - 1\right] \qquad (0.1)$$

and for $T = 0$, it is zero.

In this paper the problem of the rolling of a wheel on a rail due to inertia ($T = 0$), is solved without the assumption of similarity between the materials of the wheel and the support. Following common practice in the contact problems of the theory of elasticity, the wheel and the rail are replaced by elastic half-planes.

1. Let a wheel be moving to the right with velocity v. Since we neglect friction between the wheel and the rail and assume in advance that adhesion takes place along the whole contact region, it is clear, that no energy loss should occur and steady state can be assumed.

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* Glagolev, N.I., Issledovaniia vzaimodeistviia koles i relsov i nekotorykh sviazannykh s nim invlenii (Investigations of the interaction of wheels and rails and some phenomena connected with this interaction). *Dissertatsiia. Institut mekhaniki Akad. Nauk SSSR*, Moscow, 1948.

Using coordinates fixed at the center of the wheel and translating to the right with velocity v, we reduce the problem to the static problem of the theory of elasticity for two half-planes.

The magnitudes related to the lower half-plane (rail) will be denoted by the index *1*, to the upper (wheel) by the index 2.

For $y = 0$ we have the following boundary conditions [1]

$$v_1'(x) - v_2'(x) = \frac{x}{R}, \qquad \delta + \left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial x}\right) v = 0 \tag{1.1}$$

(along the contact line)

$$\begin{array}{ll} \sigma_y = 0, \quad \tau_{xy} = 0 & \text{outside the contact line} \\ \sigma_{y_1} = \sigma_{y_2}, \quad \tau_{xy_1} = \tau_{xy_2} & \text{along the whole line } y = 0 \end{array} \tag{1.2}$$

The following formulas express the stresses and displacements in an elastic body [1]:

$$\begin{gathered} \sigma_{y_k} + i\tau_{xy_k} = \Phi_k(z) - \Phi_k(\bar{z}) + (z - \bar{z})\overline{\Phi_k'(z)} \\ (k = 1, 2) \\ 2\mu_k(u_k' + iv_k') = \varkappa\Phi_k(z) + \Phi_k(z) - (z - \bar{z})\Phi_k(z) \end{gathered} \tag{1.3}$$

From (1.2) we obtain

$$\Phi_1(z) = -\Phi_2(z)$$

Thus from (1.3) we find

$$2\mu_1(u_1' + iv_1')_{y=0} = \varkappa_1\Phi_1^- + \Phi_1^+, \quad 2\mu_2(u_2' + iv_2')_{y=0} = -\varkappa_2\Phi_1^+ - \Phi_1^- \tag{1.4}$$

Here – and + denote the limiting values of the function Φ_1 when we approach $y = 0$ from below and from above respectively.

The two conditions (1.1) can be combined:

$$\frac{\partial u_1}{\partial x} - \frac{\partial u_2}{\partial x} + i\left(\frac{\partial v_1}{\partial x} - \frac{\partial v_2}{\partial x}\right) = \frac{\delta}{v} + i\frac{x}{R}$$

Using (1.5) we obtain boundary conditions of the problem of linear relationship for the determination of function $\Phi_1(z)$

$$\left(\frac{\varkappa_1}{\mu_1} + \frac{1}{\mu_2}\right)\Phi_1^- + \left(\frac{\varkappa_2}{\mu_2} + \frac{1}{\mu_1}\right)\Phi_1^+ = 2\left(\frac{\delta}{v} + \frac{ix}{R}\right) \tag{1.6}$$

2. The solution of the problem of linear relationship, bounded along the edges of the contact line and at infinity has the following form

$$\Phi_1(z) = 2\left(\frac{\varkappa_1}{\mu_1} + \frac{1}{\mu_2} + \frac{\varkappa_2}{\mu_2} + \frac{1}{\mu_1}\right)^{-1}\left[\frac{\delta}{v} + \frac{it}{R} - \frac{it}{R}\sqrt{(z-b)(z-a)}\left(\frac{z-b}{z-a}\right)^{i\gamma}\right] \tag{2.1}$$

where $z = b$ and $z = a$ are the coordinates of the right and left end of the contact line

$$\gamma = \frac{1}{2\pi} \ln \frac{\varkappa_1 \mu_2 + \mu_1}{\varkappa_2 \mu_1 + \mu_2} \tag{2.2}$$

For the determination of the magnitudes a, b and δ we will investigate the behavior of $\Phi_1(z)$ near the infinite point. Expanding the expression inside the brackets in (2.1) into a series, we obtain:

$$\Phi_1(z) = \left(\frac{\varkappa_1}{\mu_1} + \frac{1}{\mu_2} + \frac{\varkappa_2}{\mu_1}\frac{1}{\mu_1}\right)^{-1} \left[\frac{\delta}{v} + \frac{i(b+a)}{2R} - \right. \tag{2.3}$$
$$\left. - \gamma(b-a) + \frac{i}{2Rz}\left(\gamma^2 + \frac{1}{4}\right)(b-a)^2 + \cdots\right]$$

The function $\Phi_1(z)$ must have the following form at infinity

$$\Phi(z) = \frac{iP}{2\pi z} + 0(z) \tag{2.4}$$

Comparing (2.3) with (2.4) we obtain

$$b = -a, \qquad \frac{\delta}{v} - \frac{\gamma}{R} 2b = 0 \tag{2.5}$$
$$\left(\frac{\varkappa_1}{\mu_1} + \frac{1}{\mu_2} + \frac{\varkappa_2}{\mu_2} + \frac{1}{\mu_1}\right)^{-1} \frac{1}{R}\left(\gamma^2 + \frac{1}{4}\right) 4b^2 = \frac{P}{2\pi}$$

Thus, it turns out that the contact area is located symmetrically with respect to the center of the wheel.

For the case of plane strain $\kappa = 3 - 4\nu$; and for plane stress $\kappa = (3 - \nu)/(1 + \nu)$. Poisson's ratio for metals can be assumed to be 0.3. Thus for both cases of plane problem κ is close to two.

Consequently, the sign of γ is determined by the relationship μ_2/μ_1. In the case of a hard wheel and soft support, $(\mu_2 > \mu_1)$, γ and consequently δ, are positive. In this case $V < R\omega$.

When the support is harder than the wheel, we have $V > R\omega$.

Reynolds discovered.this phenomenon and explained it qualitatively.

The third formula in (2.5) shows that, in the case of the contact rolling of a wheel due to inertia, the length of the contact surface is somewhat smaller than for compression of two bodies of the same shape and of the same materials without friction (for $\gamma \neq 0$).

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Translated by R.M. E.-I.